

Spin and chirality orderings of the one-dimensional Heisenberg spin glass with the long-range power-law interaction

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The ordering of the one-dimensional Heisenberg spin glass interacting via the long-range power-law interaction is studied by Monte Carlo simulations. Particular attention is paid to the possible occurrence of the “spin-chirality decoupling” for appropriate values of the power-law exponent σ . Our result suggests that, for intermediate values of σ , the chiral-glass order occurs at finite temperatures while the standard spin-glass order occurs only at zero temperature.

I. INTRODUCTION

In order to understand the true nature of the experimental spin glass (SG) transition, particularly of canonical SGs which possess nearly isotropic interaction, it is crucially important to elucidate the ordering properties of the three-dimensional (3D) isotropic Heisenberg SG. Some time ago, one of the present authors (H.K.) proposed that the 3D isotropic Heisenberg SG might exhibit an intriguing “spin-chirality decoupling” phenomenon at long length and time scales, *i.e.*, the ordering of the chirality occurred at a temperature higher than the standard SG transition temperature. Chirality is a multispin variable representing the sense or the handedness of the noncoplanar spin structure induced by spin frustration. It was suggested that such a spin-chirality decoupling might play a crucial role in the experimental SG ordering [1, 2].

Concerning the possible occurrence of such spin-chirality decoupling in the 3D Heisenberg SG, however, controversy has continued some time now. Different numerical simulations by different authors reported apparently opposite conclusions [3, 4, 5, 6, 7, 8, 9, 10, 11]. Difficulty in finite-size numerical simulations might lie in the fact that, as emphasized in Ref.[4], the spin chirality-decoupling, if any, is realized only at longer length scale beyond a crossover length L^* , so that one needs to go beyond this crossover length in order to really see whether the spin-chirality decoupling occurs or not.

To understand the issue in wider perspective, it might be useful to study the phenomena by generalizing the dimensionality d from original $d = 3$ to both lower and higher dimensions. In the limit of high dimension $d \rightarrow \infty$, the model reduces to the mean-field (MF) Heisenberg SG model. In the MF limit, it has been known that there is a single SG transition at a finite temperature: There, the order parameter of the transition is the spin, not the chirality, *i.e.*, no occurrence of the spin-chirality decoupling. In $d = 1$ dimension, on the other hand, it has been known that the short-range Heisenberg SG exhibits only a $T = 0$ transition both in the spin and in the chirality. Thus, the chiral-glass phase, if any, arises in intermediate dimensions around $d = 3$. Indeed, numerical simulations for the Heisenberg SG in generalized dimensions, including $d = 2$ [12] and $d = 4, 5$ and ∞ dimensions [13], have been done recently. Though useful information was obtained from these analyses, intrinsic limitations also exist: *e.g.*, (i) the controlling parameter d cannot be changed continuously so that a fine-tuning of the phenomena was impossible, and (ii) in higher dimensions, thermalization of larger systems became increasingly difficult due to the rapid increase of the total number of spins $N = L^d$, L being the linear size of the lattice.

In order to shed further light on the issue from somewhat different perspective, we consider here a different type of Heisenberg SG model, *i.e.*, the 1D Heisenberg SG model interacting via the long-range interaction which decays with distance as a power-law with an exponent $-\sigma$. For sufficiently small σ , the model is expected to reduce to an infinite-range MF model corresponding to $d = \infty$, while, for sufficiently large σ , the model is expected to reduce to the $d = 1$ model with the short-range interaction. Hence, the variation of σ in the 1D power-law SG model might mimic that of the dimensionality d of the short-range SG model. Indeed, a recent numerical study on the corresponding 1D Ising SG model by Katzgraber and Young supported such correspondence [14].

Of particular interest here is whether the 1D Heisenberg SG with the long-range power-law interaction exhibits the spin-chirality decoupling for appropriate values of σ . In the present paper, we study by extensive Monte Carlo simulations the nature of both the spin and the chirality orderings of this model.

II. MODEL

The model we consider is the 1D classical Heisenberg model interacting via the random long-range interaction J_{ij} . The Hamiltonian is given by

$$H = - \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j, \quad (1)$$

where \vec{S}_i is the three-component classical Heisenberg spin variable, $\vec{S}_i = (S_i^x, S_i^y, S_i^z)$ with $|\vec{S}_i| = 1$. The interaction J_{ij} is assumed to obey the Gaussian distribution, decaying with distance r_{ij} as a power-law,

$$J_{ij} = C \frac{\epsilon_{ij}}{r_{ij}^\sigma}, \quad C = \sqrt{\frac{N}{\sum_{i,j} r_{ij}^{-2\sigma}}}. \quad (2)$$

where the ϵ_{ij} is chosen according to the Gaussian distribution with zero mean and the standard deviation unity:

$$\mathcal{P}(\epsilon_{ij}) = \frac{1}{\sqrt{2\pi}} \exp(-\epsilon_{ij}^2/2), \quad (3)$$

In order to make the total energy extensive, the exponent σ should be greater than 1/2.

We impose periodic boundary conditions by placing the spins on a ring. Then, the distance between the i -th and the j -th spins r_{ij} is given by [14]

$$r_{ij} = \frac{L}{\pi} \sin\left(\frac{\pi|i-j|}{L}\right), \quad (4)$$

where L is the total number of spins.

As mentioned, for sufficiently small and large σ , the model reduces to an infinite-range $d = \infty$ model and a short-range $d = 1$ model, respectively. In the limit of $\sigma \rightarrow \infty$, in particular, the model should reduce to the nearest-neighbor model where frustration is totally irrelevant. Hence, one expects that the spin-chirality decoupling would arise neither in the small- σ nor large- σ limit: It could possibly arise only for intermediate values of σ . If one notes that the dimension $d = 3$ probably lies close to the lower critical dimension of the SG order in the short-range Heisenberg SG model, the spin-chirality decoupling of the present 1D long-range SG model would arise, if any, near the borderline value of σ separating the regions of a finite-temperature SG transition and a zero-temperature SG transition.

Thanks to its one-dimensionality, some analytical results are available for the 1D long-range SG model [15, 16, 17]. On increasing σ from $\sigma = 0$, a finite-temperature SG transition changes its character from the MF one to the non-MF one beyond the borderline value of σ . This borderline value of σ , “lower critical σ ”, is known to be $\sigma = \frac{2}{3}$. In the range of $\frac{2}{3} < \sigma < 1$, the SG transition still takes place at a finite temperature, but is governed by the non-MF long-range fixed point, characterized by an “exact” SG critical-point decay exponent $\eta_{\text{SG}} = 2 - \sigma$. For σ greater than the “upper critical σ ”, $\sigma = 1$, the SG transition occurs only at zero temperature with an exponent $\eta_{\text{SG}} = 1$, which is generically expected for any zero-temperature transition with the non-degenerate ground state.

Previous analytic work did not consider the possibility of the spin-chirality decoupling [15]. If one recalls the abovementioned σ - d analogy, the spin-chirality decoupling might be expected for the range of σ around the upper critical σ , $\sigma = 1$.

III. MONTE CARLO SIMULATIONS

We perform an equilibrium MC simulation of the model. The power-law exponent σ is set to $\sigma = 1.1$, which lies in the region where we would expect the spin-chirality decoupling, if any. In our simulation, we make use of the temperature exchange MC method combined with the standard heat-bath updating. The lattice sizes studied are $L = 64, 128, 256, 512$ and 1024 , where the sample average is taken over 128-512 independent bond realizations.

The local chirality $\chi_{i\mu}$ at the i th site in the μ direction is defined by

$$\chi_{i\mu} = \vec{S}_{i+\hat{e}_\mu} \cdot (\vec{S}_i \times \vec{S}_{i-\hat{e}_\mu}), \quad (5)$$

\hat{e}_μ ($\mu = x, y, z$) being a unit lattice vector along the μ axis.

We probe the ordering of both the chirality and the spin by looking at the associated Binder ratios, *i.e.*, the spin Binder ratio g_{SG} and the chirality Binder ratio g_{CG} , as well as the associated finite-size correlation lengths, *i.e.*, the

spin correlation length $\xi_{SG}(L)$ and the chirality correlation length $\xi_{CG}(L)$. Detailed definitions of these quantities have been given in Ref.[4]. Both the Binder ratio g and the dimensionless finite-size correlation length $\xi(L)/L$ have widely been used in numerical simulations in identifying the transition point. Since both quantities are dimensionless, the data for different size L are expected to exhibit a crossing or a merging at a transition point.

In Fig.1, we show the size and temperature dependence of the Binder ratio g_{SG} (left) and of the dimensionless correlation length $\xi_{SG}(L)/L$ (right) for the spin. As can be seen from the figure, the spin Binder ratio decreases with increasing L , indicating the absence of a finite-temperature SG order. This is consistent with the result of analytical calculations showing $T_{SG} = 0$ for $\sigma > 1$ [15, 16, 17].

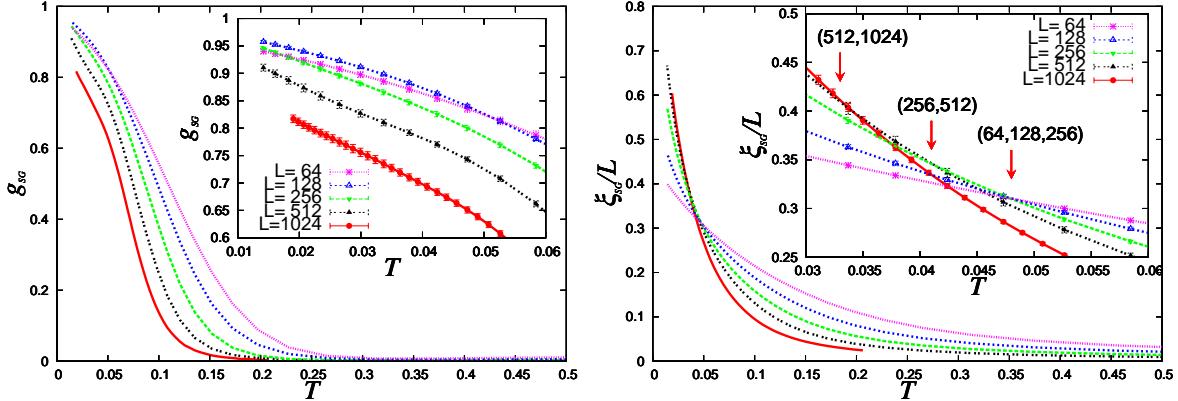


FIG. 1: The temperature and size dependence of the spin Binder ratio (left) and of the dimensionless spin correlation length (right) for $\sigma = 1.1$.

By contrast, the behavior of the dimensionless correlation length points to the opposite at first glance, *i.e.*, the data for different L appear to cross at a finite temperature $T \simeq 0.05$ suggesting the occurrence of a finite-temperature SG transition: See the main panel. A closer inspection of the data, however, has revealed that, although such a crossing indeed occurs at an almost size-independent temperature for smaller sizes $L \leq 256$, the crossing temperature rapidly shifts toward lower temperatures for larger sizes $L \geq 512$: See the inset. Such a behavior is fully consistent with the occurrence of the spin-chirality decoupling occurring beyond the crossover length scale $L^* \simeq 500$. Hence, the asymptotic behavior of the spin correlation length is eventually consistent with that of the Binder ratio and with the known analytical result.

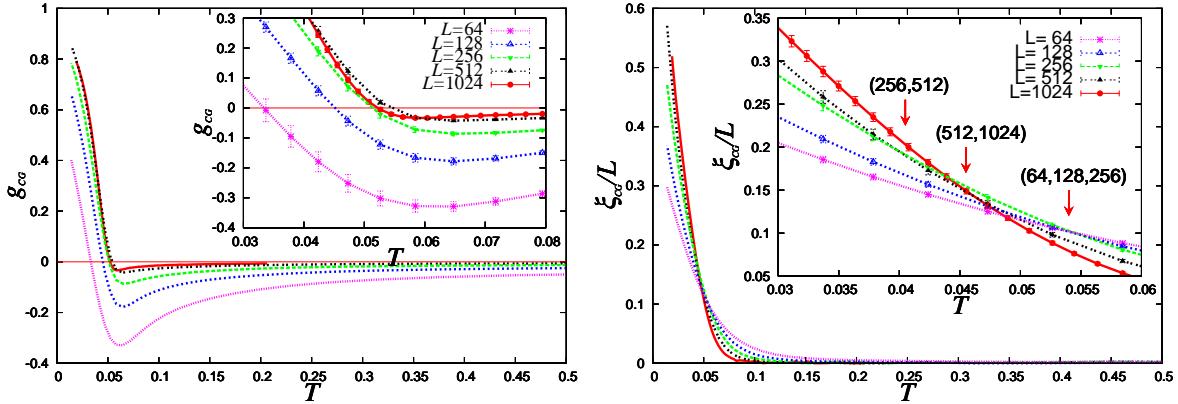


FIG. 2: The temperature and size dependence of the chirality Binder ratio (left) and of the dimensionless chirality correlation length (right) for $\sigma = 1.1$.

Next, we turn to the chirality ordering. In Fig.2, we show the size and temperature dependence of the Binder ratio g_{CG} (left) and of the dimensionless correlation length $\xi_{CG}(L)/L$ (right) of the chirality for the case of $\sigma = 1.1$. As can be seen from the figure, the Binder ratio exhibits a negative dip at a finite temperature $T = T_{dip}(L)$. The temperature $T_{dip}(\infty)$ is expected to give a chiral-glass transition temperature, which is estimated to be $T_{CG} \simeq 0.05$. This suggests

the occurrence of a finite-temperature chiral-glass (CG) transition. The dimensionless chirality correlation length exhibits a behavior similar to that of the spin correlation length, at least for the sizes $L \leq 512$. In sharp contrast to the spin correlation length, however, the crossing temperature between our two largest sizes $L = 512$ and $L = 1024$ now *shifts toward a higher temperature* as compared with the one between $L = 256$ and $L = 512$, suggesting the occurrence of a finite-temperature CG transition at $T_{\text{CG}} \simeq 0.05$, consistently with the estimate based on the chirality Binder ratio. Note that the occurrence of a finite-temperature *chiral*-glass transition at $\sigma = 1.1$ does not contradict any known analytical result on this model.

The combined data for the spin and for the chirality give a fairly strong numerical support for the occurrence of the spin-chirality decoupling in the present model, *i.e.*, $T_{\text{CG}} \simeq 0.05$ and $T_{\text{SG}} = 0$. This decoupling becomes clear only after studying larger lattices $L \gtrsim 500$, suggesting that the crossover length scale in this model might be rather large, $L^* \simeq 500$, presumably reflecting the long-range nature of the interaction.

IV. SUMMARY AND DISCUSSION

By numerically investigating the spin and the chirality orderings of the one-dimensional Heisenberg SG with the long-range power-law interaction for the case of $\sigma = 1.1$, we observed a strong numerical evidence that the CG transition occurs at a finite temperature while the standard SG transition occurs only at zero temperature, *i.e.*, the occurrence of the spin-chirality decoupling. If one believes the $\sigma - d$ analogy, our present observation may give some support to the occurrence of the spin-chirality decoupling in the original $d = 3$ -dimensional Heisenberg SG.

We wish to emphasize again that the spin-chirality decoupling has become eminent when one studies larger systems. This is particularly so when one looks at the correlation lengths: Remember that the data of correlation lengths for smaller lattices $L \lesssim 500$ spuriously suggested the occurrence of a simultaneous spin and chiral transition at a finite temperature, *i.e.*, the absence of the spin-chirality decoupling, which, however, contradicted the analytical result on this model. Namely, if at $\sigma = 1.1$ there were a simultaneous spin and chiral transition at a finite-temperature without the spin-chirality decoupling, the standard RG analysis should apply, inevitably yielding the exponent relation $\eta_{\text{SG}} = 2 - \sigma = 0.9$. Since the SG correlation function at finite T_{SG} decays with distance r as $r^{-(\eta-1)}$, however, this leads to an immediate contradiction. Therefore, a simultaneous spin and chiral transition without the spin-chirality decoupling as apparently suggested from the correlation-lengths data for smaller sizes $L \lesssim 500$ is not allowed at $\sigma = 1.1$.

It is also a bit surprising that the spin Binder ratio and the normalized spin correlation length exhibit quite different behaviors for moderate lattice sizes $L \lesssim 500$. While the absence of the standard SG order is evident in the spin Binder ratio already from rather small lattices, it becomes appreciable in the corresponding spin correlation length only for larger lattices, say, $L \gtrsim 500$. In this connection, it is sometimes mentioned in the literature that the correlation length might be the best quantity to look at in the study of phase transition, being superior to, *e.g.*, the Binder ratio [7]. However, our result indicates that this is not always the case: In the present occasion, on the contrary, the Binder ratio is a better quantity than the correlation length, at least for moderate lattice sizes. Of course, both quantities have given the same conclusion for large enough lattices, as it should be.

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